24/10/2015

سكشن [٤] رياضة

AMR

M.r

[٥]

Elementary f_n

① الدالة اللوغارتمية

$$F(z) = \ln(z) = \ln(r) + i\theta$$

$$= \ln(r) + i(\theta + 2n\pi)$$

↳ multi value

1. continous
2. analytic
3. not Periodic
4. multi _ value

Note. ← أي دالة عكسية $(\ )^{-1}$ (الكسرية)

$z^{1/n}$ هتوصلك الـ (Ln)

وبالتالي نفس خصائص (Ln)!!

[2] الدالة الأسية

$$F(z) = e^{z}$$

1. continous
2. analytic
3. periodic $2\pi i$
4. single _ value

[3] الدوال المثلثية

1. continous
2. analytic
3. Periodic 2π
4. single value.

* evaluete

[1] $\ln(z) = \ln(\sqrt{2} + i\sqrt{6})$

↳ $r = \sqrt{2+6} = \sqrt{8} =$

$\theta = \tan^{-1}\frac{\sqrt{6}}{\sqrt{2}} = \pi/3 = 60$

$$\therefore \ln(z) = \ln \sqrt{2}\, e^{\pi/3\, i}$$

$$\ln(z) = \ln\sqrt{8} + i(\pi/3)$$

→ 60 خواصها خواص ln

(16)

(2) $3^{i} = e^{\ln 3^{i}} = e^{i\ln 3}$

$= \cos(\ln 3) + i\sin(\ln(3))$

• نفس خواص الدوال المثلثية

(3) $e^{z} = z$

$\ln e^{z} = \ln z$

$z = \ln z + i(0)$

نفس خواص ln z

show that $\frac{d}{dz}(\ln(z)) = \frac{1}{z}$

$\ln(z) = \ln re^{i\theta} = \ln r + i\theta$

$u = \ln r$, $v = \theta$

$u_r = \frac{1}{r}$ $= \frac{1}{r}$ $u_\theta = 0$

$v_r = 0$ $v_\theta = 1$

• Cuchy تحقق شروط

• الدالة analytic

معلومة

$F'(z) = e^{-i\theta}(u_r + iv_r)$

$F'(z) = e^{-i\theta}\left(\frac{1}{r} + i0\right)$

$F'(z) = \frac{1}{re^{i\theta}} = \frac{1}{z}$ ※

C

* show that

$$(1+i)^{i} = e^{-(\pi/4 \pm 2n\pi)} \cdot e^{\frac{i}{2}(\ln(2))}$$

L.H.S $\rightarrow$ $e^{\ln(1+i)^{i}} = e^{i\ln(1+i)} = e^{i(\ln(\sqrt{2}) + i(\pi/4 + 2n\pi))}$

$\Rightarrow \ln(1+i) = \ln(\sqrt{2}) + i\,\pi/4$
$\hookrightarrow r = \sqrt{2}$
$\hookrightarrow \theta = \pi/4$

$$= e^{i\ln(\sqrt{2})} \cdot e^{-(\pi/4 + 2n\pi)}$$

$$= e^{i\ln 2^{\frac{1}{2}}} \cdot e^{-(\pi/4 + 2n\pi)}$$

$$= e^{\frac{i}{2}\ln(2)} \cdot e^{-(\pi/4 + 2n\pi)} = \text{R.H.S}$$

* $\sin Z = \dfrac{e^{iz} - e^{-iz}}{2i}$, $\cos Z = \dfrac{e^{iz} + e^{-iz}}{2}$

$\sinh Z = \dfrac{e^{z} - e^{-z}}{2}$, $\cos Z = \dfrac{e^{z} + e^{-z}}{2}$

$$\cosh^2 - \sinh^2 = 1$$

$$\sinh(A+B) = \sinh(A)\cosh(B) + \cosh(A)\sinh(B)$$

$$\cosh(A+B) = \cosh(A)\cosh(B) + \sinh(A)\sinh(B)$$

(d)

الأفكار

١ إيجاد علاقة جبرية .

٢ فك الدوال المثلثية .

٣ دالة مثلثية عكسية .

II show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$

L.H.S $|\sin z|^2 =$

$\sin z = \sin(x+iy) = \sin x \cos iy + \cos x \sin iy$

$= \sin x \cosh y + i \cos x \sinh y$

$\therefore |\sin z|^2 = (\sin x \cosh y + i\cos x \sinh y)\cdot(\sin x \cosh y - i \cos x \sinh y)$

$= \sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y$

$\cosh^2 y \to (1+\sinh^2 y)$, $\cos^2 x \to (1-\sin^2 x)$

$= \sin^2 x + \sin^2 x \sinh^2 y + \sinh^2 y - \sin^2 x \sinh^2 y$

$= \sin^2 x + \sinh^2 y$ ⁂

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e

(8) solve $\cosh z = \frac{1}{2}$ → $z = ??$

$$\frac{e^{z} + e^{-z}}{2} = \frac{1}{2}$$

$\therefore$ $e^{z} + e^{-z} = 1$ بالضرب * e^{z}

$$e^{2z} + 1 = e^{z}$$

$$e^{2z} - e^{z} + 1 = 0$$

$\therefore$ $e^{z} = \frac{+1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$$z = \ln\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\right)$$

$z = \ln\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$

$r = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$z = \ln 1 + i\left(\frac{\pi}{3}\right)$

or $z = \ln 1 + i\left(\frac{\pi}{3}\right) + 2\pi n$

$z = \ln\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

$r = 1$

$\theta = 2\pi - \frac{\pi}{3}$

$z = \ln 1 + i\left(2\pi - \frac{\pi}{3}\right)$

F

* show that $\sinh^{-1} z = \ln\left(z + \sqrt{z^2+1}\right)$

let $w = \sinh^{-1} z$

$$z = \sinh w = \frac{e^{w} - e^{-w}}{2}$$

$2z = e^{w} - e^{-w}$ بالضرب $* e^{w}$

$$2z e^{w} = e^{2w} - 1$$

$$e^{2w} - 2z e^{w} - 1 = 0$$

$$e^{w} = \frac{2z \pm \sqrt{4z^2 - 4}}{2}$$

$$e^{w} = z \pm \sqrt{z^2 + 1}$$

$$\ln e^{w} = w = \ln\left(z \pm \sqrt{z^2+1}\right)$$

$$\sinh^{-1} z = \ln\left(z \pm \sqrt{z^2+1}\right)$$

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